Enhancing Forecast Accuracy for Damped Trend and Multiplicative Seasonality: A Hybrid Time-Series Framework with Simulation Study

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Abstract

Forecasting time series characterized by a damped trend and multiplicative seasonality (DTMS) presents a significant challenge, as conventional models like ARIMA and ETS often fail to capture their complex, non-linear interactions. This study bridges this gap by developing and evaluating a novel hybrid forecasting framework that synergistically combines statistical and machine learning (ML) approaches. We hypothesize that while statistical models capture linear components, ML models excel at modeling non-linear residuals. Our methodology employs a comprehensive simulation study to systematically control data characteristics, alongside an empirical analysis of a real financial dataset from the Dhaka Stock Exchange. Results demonstrate that hybrid models, particularly SVR-ANN and SVR-ETS, significantly outperform individual and other hybrid models across all accuracy metrics (RMSE, MAE, MAPE, MASE). The simulation confirmed this superiority, with the top hybrids achieving a mean MASE of 0.701. This indicates that an ML model, especially SVR, is highly effective in modeling the complex residuals from a primary statistical forecast. We conclude that a hybrid paradigm integrating statistical and ML techniques offers a robust and superior solution for forecasting DTMS series, providing practitioners with evidence-based guidance for enhanced accuracy.

Keywords: Dhaka Stock Exchange; Damped trend and multiplicative seasonality; Hybrid machine learning; Low-volatility regime; Support Vector Regression.

1. Introduction

Forecasting time series that exhibit a damped trend—a pattern where growth decelerates over the forecast horizon—combined with multiplicative seasonality presents a significant challenge in fields such as finance, supply chain management, and energy demand planning. While damped trend models, originally introduced by Gardner and McKenzie in 1985, provide a useful framework by reducing trend effects over longer horizons [1-4], they do not fully address the complexities introduced by multiple or changing seasonal cycles. Usually, the damped trend model is estimated by using a structural model based on the Kalman filter [5,6] or by adopting the innovations approach. In the innovations approach usually different types of hybrid models were developed for time-series forecasting [7,8]. The concomitant presence of a damped trend and multiplicative seasonality (DTMS) presents a significant challenge, frequently exceeding the capabilities of conventional forecasting models, including Autoregressive Integrated Moving Average (ARIMA) and Exponential Smoothing State Space (ETS) models. These models excel with linear trends and stable seasonality but frequently

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struggle to capture the nuanced, non-linear interactions inherent in such complex data. These shortcomings reveal a pronounced void in the literature, highlighting the demand for advanced analytical frameworks capable of delivering enhanced accuracy for series exhibiting DTMS characteristics.

Existing models like ETS and ARIMA are widely used for their simplicity and effectiveness in handling linear trends and additive seasonality. However, these classical methods often underperform when confronted with nonlinear patterns and multiplicative seasonal effects. More advanced models, such as the Trigonometric, Box-Cox, ARMA errors, Trend, Seasonal (TBATS) framework and Artificial Neural Networks (ANN), have been developed to better manage complex seasonal structures and nonlinearities [9,10]. Despite this progress, there remains a gap in understanding which combinations of statistical and machine learning (ML) methods offer the most reliable forecasts under varying conditions of damped trends and multiplicative seasonality. Hybrid models, including ANN-ARIMA [11], ETS-ANN [12], TBATS-ARIMA, TBATS-ETS, and TBATS-ANN, combine classical models' robustness with machine learning methods for complex time-series data prediction, offering superior performance in large datasets with seasonal patterns [13-15].

Therefore, this study aims to bridge this gap by developing and rigorously evaluating a novel hybrid forecasting framework that synergistically combines statistical and ML approaches. We hypothesize that while statistical models can effectively capture the linear components (trend, seasonality) of a time series, ML models are better suited to model the complex, non-linear residuals that remain. By integrating these paradigms, we aim to achieve superior forecast accuracy for series with DTMS feature. We incorporate Seasonal-Trend decomposition using LOESS (STL) as a preprocessing step for seasonal-adjusted models (STL-ARIMA, and STL-ETS) to handle multiplicative seasonality. We conduct a comprehensive, simulation-based comparison of a wide array of models, from individual statistical and ML techniques to their hybrid combinations (ETS-ARIMA, ETS-TBATS, TBATS-ETS, TBATS-ARIMA and TBATS-ANN). Unlike studies reliant solely on real-world data, we employ a simulation study to systematically control data characteristics, allowing us to precisely identify which models excel under specific seasonal and trend conditions. Finally, we provide clear, evidence-based guidance on model selection and sequencing for practitioners facing this common yet complex forecasting scenario.

2. Method and Methodology

2.1 Data Description

The study data contains information about the monthly average closing price (which is simply the arithmetic mean of all the daily closing prices for that specific month) in Bangladeshi Taka (BDT) of Renata Pharmaceuticals PLC from Dhaka Stock Exchange, covering the period from January 2000 to December 2023.

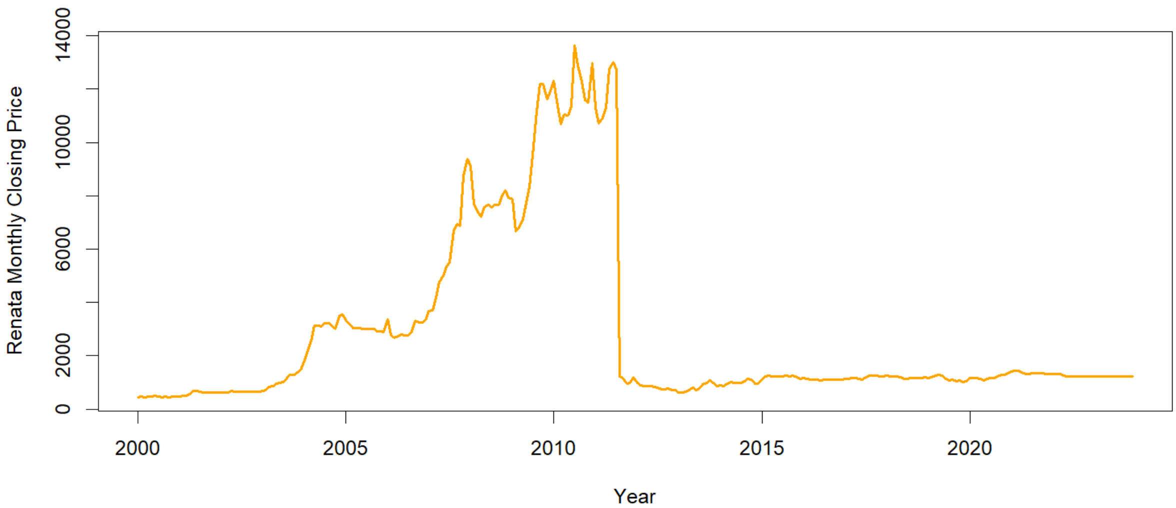

Figure 1: Monthly average closing price of Renata Pharmaceuticals PLC from Jan 2000 to Dec 2023.

Figure 1 shows that the Renata Pharmaceuticals PLC's monthly average stock price has experienced several periods of growth and volatility. The early growth phase saw minimal volatility, indicating a stable phase. From 2004 to 2011, the stock price experienced a sharp and sustained increase, reaching several peaks. The dramatic rise and volatility may be attributed to strong business growth, market optimism, or sectoral developments. In 2011, a sudden price drop occurred, falling from above 12,000 Taka to below 2,000 Taka almost instantly. The long-term stability of the stock price from 2012 to 2023 suggests a stable price, indicating a mature and possibly less volatile stock. Therefore, the monthly closing price of Renata Pharmaceuticals PLC represents a DTMS nature of time series data.

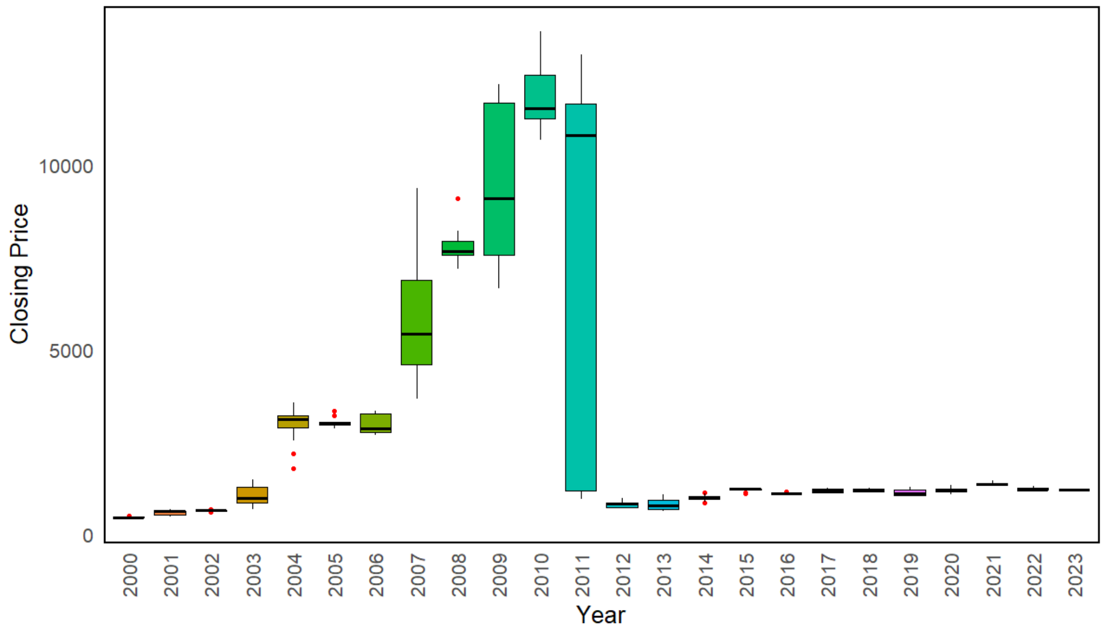

Figure 2: Yearly boxplot of Renata Pharmaceuticals PLC's monthly closing prices (2000–2023). Each box represents the distribution of monthly closing prices within a year. The box bounds the interquartile range (IQR), the horizontal line marks the yearly median, whiskers extend 1.5 × IQR, and red points indicate outliers. Colors differentiate the years.

The boxplots in Figure 2 shows Renata Pharmaceuticals PLC's stock price trajectory over time. From 2000-2005, prices were low, indicating limited volatility. From 2006, prices soared, with medians exceeding 10,000 BDT and widening interquartile ranges. This period was marked by speculative activity and price swings. After 2011, prices declined, and volatility diminished. By 2015-2023, distributions stabilized, indicating a post-boom correction phase and a shift towards a lower valuation regime. Figure 2 also emphasizes the DTMS scenarios, as does Figure 1.

2,2 Data Preprocessing

Following data preprocessing, which involved the identification and correction of outliers and imputation of missing values, we explored the seasonal effect on the monthly closing price of Renata Pharmaceuticals PLC (in BDT) using the STL decomposition method [16,17]. Figure 3 displays the original monthly closing price (navy line) alongside the seasonal-adjusted (magenta line) and trend components (teal line). The seasonal-adjusted line closely follows the original series, indicating that seasonal fluctuations are present but relatively small. The trend line captures the major price movements, including sharp rises and falls, underscoring that trend is the dominant driver of price changes.

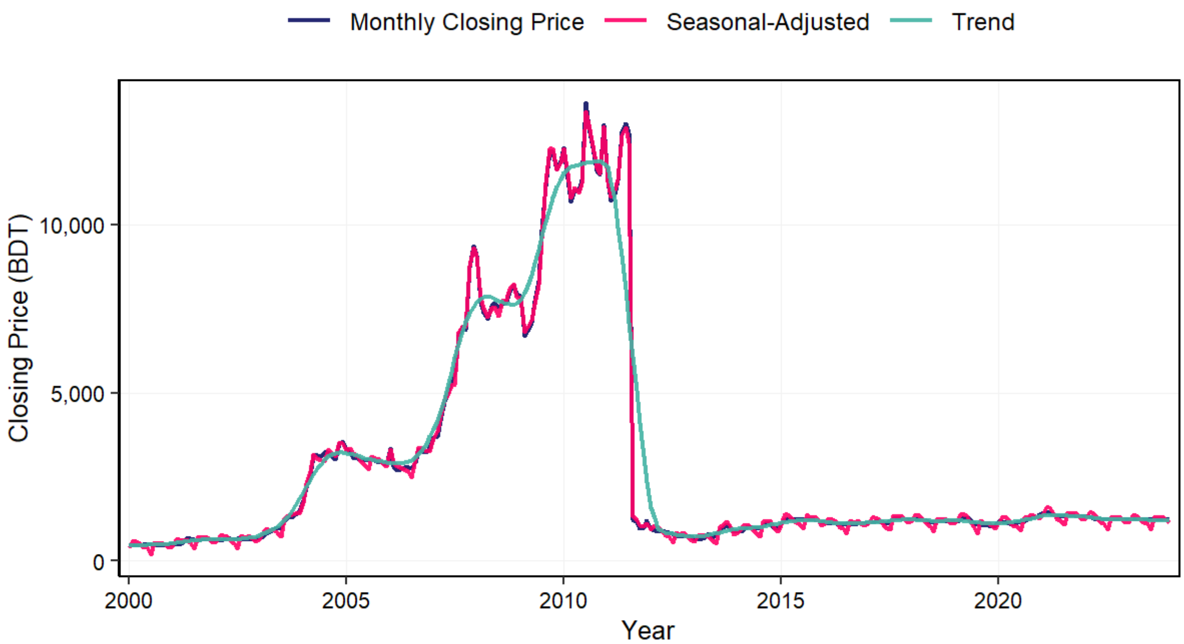

Figure 3: Monthly closing price of Renata Pharmaceuticals PLC (2000–2023) with seasonal adjustment and trend component. The plot displays the raw monthly closing prices (navy), seasonally adjusted prices (magenta), and the long-term trend (teal).

In Figure 4, the ridgeline density plot of monthly prices over the years, shows some variation in price distributions across months—evidence of seasonal effects. However, the overlaps and spread of ridges imply that these seasonal differences are subtle rather than stark or highly variable. The circular heatmap of average monthly closing prices in Figure 5, reveals some variation in average prices across months, indicating seasonality. Yet the color differences between months are modest: while July and January show slightly higher and lower averages, the range of seasonal variation is narrow compared to the overall price scale.

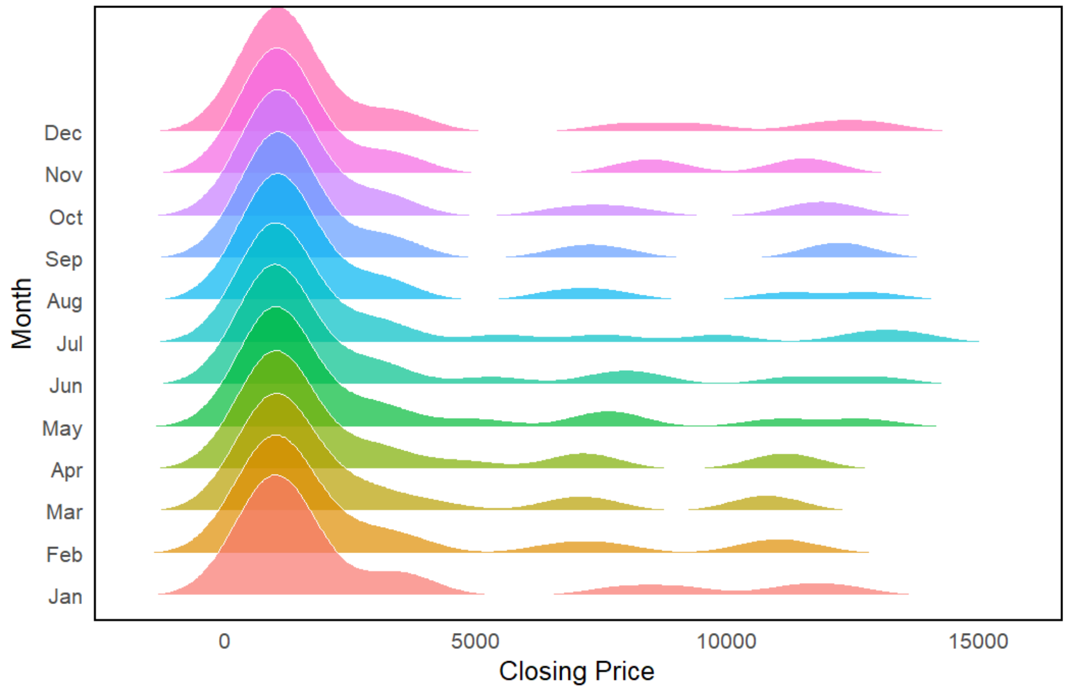

Figure 4: Ridgeline density of Renata Pharmaceuticals PLC's monthly closing prices (2000–2023). Each ridge represents the distribution of stock prices across all years for a given month. The spread and peaks indicate the relative frequency and volatility of closing prices over time.

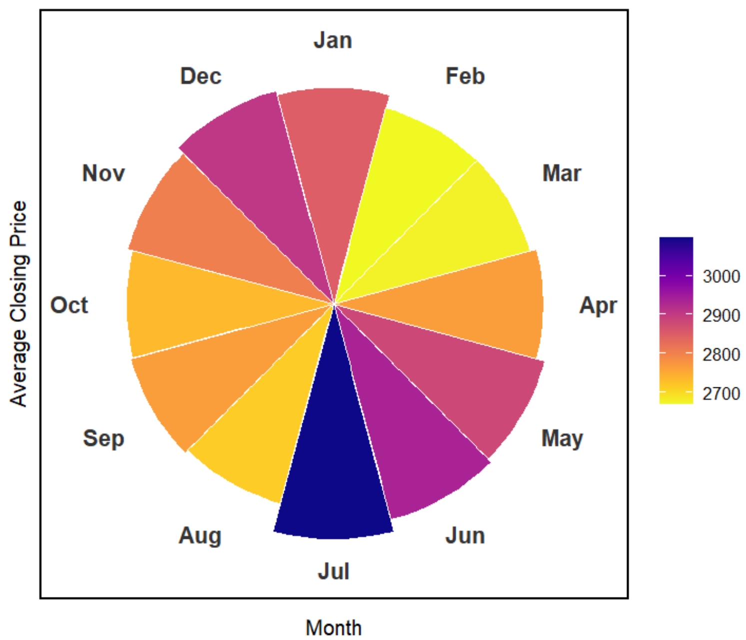

Figure 5: Circular heatmap of Renata Pharmaceuticals PLC's average monthly closing prices (2000–2023). Each sector represents a month of the year, with colors reflecting the average closing price level. The color gradient ranges from yellow (lower average values) to dark blue (higher average values).

Figures 3-5 illustrate that while multiplicative seasonality is present in the data, its influence is relatively mild and overshadowed by the more dominant and impactful long-term trends in the monthly closing prices. This suggests modeling approaches should emphasize capturing trend dynamics alongside seasonality to improve forecasting accuracy. To explicitly emphasize the capture of trend and seasonal components, our analysis incorporated seasonally-adjusted variants of individual models (e.g., STL-ARIMA, STL-ETS, STL-ANN, STL-SVR). Hybrid models were not evaluated in a seasonally-adjusted framework to maintain architectural clarity. The TBATS model was also not combined with STL, as its trigonometric formulation inherently accounts for complex seasonality, making additional adjustment redundant.

Subsequent to data preprocessing, the dataset was divided into a training set (70%) for model fitting and a test set (30%) for performance evaluation. It is critical to note that seasonal adjustment was not a universal step; it was applied as a targeted preprocessing stage only for those specific models designed to incorporate it (namely, the STL-ARIMA, STL-ETS, STL-ANN, and STL-SVR frameworks). Forecasting performance was evaluated across a suite of models designed to capture damped trends and multiplicative seasonality. This included statistical time-series models (ARIMA, SARIMA, ETS, TBATS), ML models (ANN, SVR), and their enhanced variants: seasonally-adjusted frameworks and hybrid architectures that integrate statistical and machine learning components.

2.3 Time Series Models

This study evaluates several time series modeling approaches for capturing and forecasting the dynamics of a damped trend with multiplicative seasonality. The models under consideration are as follows:

Autoregressive Integrated Moving Average Model (ARIMA)

We employ ARIMA models to capture linear dependencies between observations and past values, with differencing applied to achieve stationarity [18]. The Box-Jenkins method is widely used to determine appropriate values for p, d, and q. The ARIMA (p, d, q) process can be expressed as a differential equation where $\{X_t\}$ satisfies

$$\phi(B)(1-B)^d X_t = \theta(B) Z_t, \quad (1)$$

with $\{Z_t\} \sim WN(0, \sigma^2)$ and $\phi(B)$and $\theta(B)$ are the polynomials of backshift operator B with degrees p and q, respectively [19].

Seasonal Autoregressive Integrated Moving Average Model (SARIMA)

The SARIMA model captures linear relationships within the data by integrating several components: autoregressive and moving average terms to account for serial dependence, differencing to achieve stationarity, and seasonal parameters to model periodic fluctuations. $\{X_t\}$ is a seasonal $\text{ARIMA}(p,d,q)\times(P,D,Q)^s$ process with period s and nonnegative differencing integers d and D, can be expressed as

$$\phi(B)\Phi(B^s)(1-B)^d(1-B^s)^D X_t=\theta(B)\Theta(B^s)Z_t, \tag{2}$$

where $\{Z_t\}\sim WN(0,\sigma^2)$. The polynomial $\Phi(B^s)$ and $\Theta(B^s)$ are defined as $(1-\Phi_1B^s-\Phi_2B^{2s}-\cdots-\Phi_pB^{ps})$ and $(1+\Theta_1B^s+\Theta_2B^{2s}+\cdots+\Theta_pB^{qs})$, respectively [19].

Exponential Smoothing State Space (ETS) Models

The ETS framework is a statistical method used to model time series by decomposing them into error, trend, and seasonal components. Its state space formulation aids in estimating smoothing parameters and generating probabilistic forecasts. Innovations state space models are a general class of exponential smoothing models, defining the interaction between these components. ETS (M, A_d, M) model with multiplicative errors, additive damped trend, and additive seasonality can be defined as

$$\begin{aligned}
y_t &= (l_{t-1}+\phi b_{t-1})s_{t-m}(1+\varepsilon_t)\\
l_t &= (l_{t-1}+\phi b_{t-1})(1+\alpha\varepsilon_t)\\
b_t &= \phi b_{t-1}+\beta(l_{t-1}+\phi b_{t-1})\varepsilon_t\\
s_t &= s_{t-m}(1+\gamma\varepsilon_t),
\end{aligned} \tag{3}$$

where l_t, b_t and s_t are level, trend and seasonality equations, respectively, and α,β,γ and ϕ for level, trend, seasonality and damped trend parameters, respectively. Moreover, we assume multiplicative errors with $\varepsilon_t\sim \text{NID}(0,\sigma^2)$ [20].

Trigonometric Seasonality, Box-Cox transformation, ARMA errors, Trend, Seasonal Components (TBATS) Model

TBATS model is specifically designed to handle complex seasonal patterns and irregularities. It combines a Box-Cox transformation to stabilize variance, trigonometric functions to parsimoniously model multiple seasonality, and ARMA error processes to account for residual autocorrelation, in addition to damped trend components. $\text{TBATS}(\omega,\{p,q\},\varphi,\{<m_1,k_1>,<m_2,k_2>,\ldots,<m_T,k_T>\})$, where, ω is a Box-Cox transformation, p, q is ARMA parameters, φ is a damping parameter, $m_{1,\ldots,}m_T$ are seasonal periods, $k_{1,\ldots,}k_T$ are the number of fourier series pairs. The model can be written as

$$y_t^{(\omega)}=l_{t-1}+\varphi b_{t-1}+\sum_{i=1}^{T}s_{t-1}^{(i)}+\alpha d_t$$

$$b_t = b_{t-1} + \beta d_t$$

$$s_t^{(i)} = \sum_{j=1}^{k_i} s_{j,t}^{(i)} \tag{4}$$

$$s_{j,t}^{(i)} = s_{j,t-1}^{(i)} \cos\lambda_j^{(i)} + s_{j,t-1}^{*} \sin\lambda_j^{(i)} + \gamma_1^{(i)} d_t$$

$$s_{j,t}^{*(i)} = -s_{j,t-1}^{(i)} \sin\lambda_j^{(i)} + s_{j,t-1}^{*} \cos\lambda_j^{(i)} + \gamma_2^{(i)} d_t$$

$$\lambda_j^{(i)} = \frac{2\pi j}{m_i},$$

where $i = 1, \ldots, T$, d_t is an ARMA (p, q) process, α, β, γ_1 and γ_2 are smoothing parameters, l_0 is an initial level, and b_0 is a slope value [10].

2.4 Machine Learning (ML) Time Series Models

Artificial Neural Network (ANN) Model

ANNs are a highly effective alternative to ARIMA for time series forecasting, primarily due to their ability to model complex, nonlinear patterns without prior assumptions about the data's structure [11]. A typical ANN model for forecasting relates an output y_t to a set of p lagged inputs $\left(y_{t-1}, y_{t-2}, \ldots, y_{t-p}\right)$ using a single hidden layer with q neurons and a sigmoid activation function $g(x) = \frac{1}{1+e^{-x}}$. The model is defined by

$$y_t = \alpha_0 + \sum_{j=1}^{q} \alpha_j g\ \left(\beta_{0j} + \sum_{i=1}^{p} \beta_{ij} y_{t-1}\right) + \varepsilon_t, \tag{5}$$

where α_j represents the weight between j^{th} represents output neuron bias, β_{0j} represents hidden neuron bias, β_{ij} represents weight between i^{th} input and j^{th} hidden neuron, p is inputs, and q is hidden neurons, also the sigmoid activation function is applied to the hidden layer [11,21].

Support Vector Regression (SVR) Model

SVR is a powerful supervised machine learning algorithm derived from the principles of Support Vector Machines (SVMs) for solving regression problems. It is particularly well-suited for time series forecasting due to its exceptional ability to model complex, non-linear relationships inherent in temporal data. The core objective of SVR is to find a function $f(\mathbf{x})$ that deviates from the actual target values y_n by no more than a predefined margin of error ε for all training data, while simultaneously being as flat as possible. This is achieved using an ε-insensitive loss function, which does not penalize predictions within the ε-tube [22]. A key strength of SVR is its robustness to outliers, as the model's fit is determined only by a subset of critical data points called support vectors, leading to more generalized forecasts. For a non-linear problem, the input data $\mathbf{x}$ is mapped to a higher-dimensional feature space using a kernel function $K\left(\mathbf{x}_i, \mathbf{x}_j\right)$. The general forecasting equation for SVR is given by

$$f(\mathbf{x}) = \sum_{n=1}^{N} (\alpha_n - \alpha_n^*) K(\mathbf{x}_n, \mathbf{x}) + b, \tag{6}$$

where $f(\mathbf{x})$ is the predicted value for the input vector $\mathbf{x}$; α_n, and α_n^* are Lagrange multipliers satisfying $0 \le \alpha_n, \alpha_n^* \le C$; most are zero, and the non-zero weights define the support vectors; b is the bias term; and $C > 0$ is a regularization parameter that controls the trade-off between the flatness of f and the amount by which deviations larger than ε are tolerated [22].

2.5 Hybrid ML-based Model's Framework

Our proposed hybrid modeling approach directly integrates time series and machine learning techniques on data exhibiting damped trend and multiplicative seasonality, foregoing an initial seasonal adjustment step. The framework for this hybrid construction is formalized in Algorithm 1, which is adapted from the methodology proposed by [23].

Algorithm 1: Hybrid ML-based Forecasting for DTMS data

Input: Original DTMS time series data $Y = \{y_1, y_2, \cdots, y_T\}$

Output: Final forecast $\hat{F}_{final}$

1: First-Stage Modeling:

- Fit a primary time series (TS) or ML model M_1 to the original DTMS data Y.
- Obtain the first-stage forecasts $\hat{F}_{M1}$ and calculate the residuals $\hat{E} = Y - \hat{F}_{M1}$.

2: Second-Stage (Residual) Modeling:

- Fit a secondary model M_2 (another ML or TS model) to the residual series $\hat{E}$.
- Obtain the forecasts $\hat{F}_{M2}$ of the residuals $\hat{E}$. .

3: Forecast Aggregation:

- Calculate the final hybrid forecast by combining the first-stage forecast and the predicted residuals: $\hat{F}_{final} = \hat{F}_{M1} + \hat{F}_{M2}$.

2.6 Forecasting Accuracy Metrics

The forecasting performance of the hybrid models was evaluated using four key metrics: Mean Absolute Error (MAE), Root Mean Squared Error (RMSE), Mean Absolute Percentage Error (MAPE), and Mean Absolute Scaled Error (MASE). These metrics were calculated as follows:

$$\text{MAE} = \frac{1}{n} \sum_{i=1}^{n} |y_i - \hat{y}_i|, \tag{7}$$

$$\text{MAPE} = \frac{1}{n} \sum_{i=1}^{n} \frac{|y_i - \hat{y}_i|}{y_i} \times 100\%, \tag{8}$$

$$\text{MASE} = \frac{1}{n}\sum_{i=1}^{n}\left(\frac{\left|y_i - \hat{y}_i\right|}{\frac{1}{n-m}\sum_{j=m+1}^{n}\left|y_j - y_{j-m}\right|}\right), \tag{9}$$

$$\text{RMSE} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(y_i - \hat{y}_i\right)^2}, \tag{10}$$

where n indicates the number of observations, y_i denotes the original values, and $\hat{y}_i$ represents the predicted values, and m is the seasonality value [24]. The scale-dependent metrics MAE and RMSE are derived from squared errors and absolute errors, respectively. MASE is a scale-free error metric, while MAPE is a unit-free error measure based on percentage errors.

3. Results and Analysis

3.1 Real Data Analysis

We assessed the forecasting performance of multiple model classes—including classical time series, ML, seasonally adjusted, and hybrid approaches—on a real-world financial dataset characterized by a DTMS. The models were applied to the monthly closing price of Renata Pharmaceuticals PLC for the period January 2000 to December 2023. A comparative analysis of their results is presented in Table 1. The forecasting performance, as detailed in Table 1, reveals that the hybrid models SVR-ANN and SVR-ETS were clearly superior. This is evidenced by their lowest error metrics across all evaluated measures (SVR-ANN: RMSE=36.45, MAE=24.04, MAPE=1.98%, MASE=1.066; SVR-ETS: RMSE=36.47, MAE=24.47, MAPE=2.02%, MASE=1.084). Notably, both models achieved a MASE value below 1, indicating that they forecast more accurately on average than a naive seasonal benchmark model, a gold standard for forecasting accuracy. The standalone SVR model (RMSE=37.98, MAE=27.42, MAPE=2.26%, MASE=1.215) and the SVR-ARIMA hybrid also performed exceptionally well (RMSE=37.98, MAE=27.42, MAPE=2.26%, MASE=1.215). The dominance of models leveraging SVR suggests it is exceptionally powerful at modeling the monthly DTMS patterns inherent in this specific financial time series.

Table 1: Forecasting accuracy measures of different classical time-series, ML, seasonal adjusted, and hybrid models for predicting DTMS series data.

Model	MSE	RMSE	MAE	MAPE	MASE
ETS	7782.3881	88.2178	66.4304	5.4652%	2.9445
SARIMA	7782.3799	88.2178	66.4301	5.4651%	2.9445
TBATS	7777.8796	88.1923	66.2456	5.4476%	2.9363
ANN	70229.7419	265.0089	247.2681	19.7934%	10.96
SVR	1442.8799	37.9853	27.4276	2.2602%	1.2157
STL-ARIMA	6549.7719	80.9307	61.3176	5.0039%	2.7179
STL-ETS	6546.4722	80.9103	61.527	5.0278%	2.7271

STL-SVR	172659.605	415.5233	261.5777	21.8975%	11.5943
STL-TBATS	6549.8307	80.931	61.3167	5.0038%	2.7178
STL-ANN	1453319.15	1205.5369	898.9924	73.2752%	39.8473
ARIMA-SVR	7506.3329	86.6391	66.7093	5.5083%	2.9569
ETS-SVR	7782.4702	88.2183	66.4325	5.4654%	2.9446
TBATS-SVR	7778.1349	88.1937	66.2565	5.4487%	2.9368
ANN-SVR	64858.9233	254.6742	239.2084	19.2068%	10.6028
ETS-ARIMA	7782.537	88.2187	66.4349	5.4656%	2.9447
TBATS-ARIMA	7777.8796	88.1923	66.2456	5.4476%	2.9363
ANN-ARIMA	70229.7419	265.0089	247.2681	19.7934%	10.96
SVR-ARIMA	1442.8799	37.9853	27.4276	2.2602%	1.2157
ARIMA-ETS	7803.3845	88.3368	67.0872	5.5273%	2.9736
SVR-ETS	1330.4484	36.4753	24.4765	2.0204%	1.0849
TBATS-ETS	7777.9083	88.1925	66.2469	5.4478%	2.9364
ANN-ETS	70145.0712	264.8491	247.0969	19.7794%	10.9524
ARIMA-ANN	15264.8011	123.5508	104.4561	8.9035%	4.63
ETS-ANN	7782.2074	88.2168	66.4631	5.4683%	2.9459
TBATS-ANN	7778.1833	88.194	66.2542	5.4484%	2.9367
SVR-ANN	1329.1468	36.4575	24.0498	1.9852%	1.066
ARIMA-TBATS	14470.1676	120.292	98.2027	7.7397%	4.3528
ETS-TBATS	7782.4681	88.2183	66.4312	5.4652%	2.9445
SVR-TBATS	6497.6734	80.6081	73.8175	6.0459%	3.2719
ANN-TBATS	74615.0443	273.1575	258.0678	20.6949%	11.4387

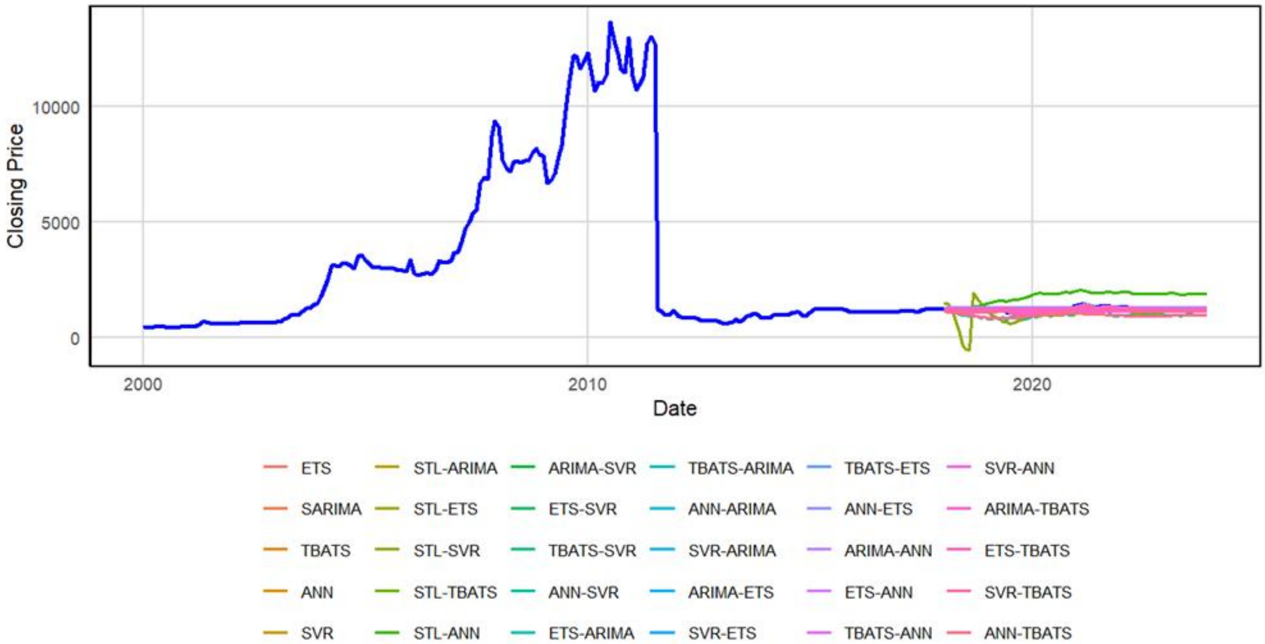

Figure 6: Forecasts for the monthly closing price (in BDT) of Renata Pharmaceuticals PLC using different classical time-series models, ML models, seasonal-adjusted models and ML-based hybrid models.

This quantitative assessment is unequivocally supported by the visual diagnostics presented in Figure 6. The plot of forecasted versus actual monthly closing prices (in BDT) reveals that the SVR-ANN model provides the most coherent fit, effectively capturing the series' DTMS with minimal deviation. Furthermore, the fact that the top performers all incorporate SVR-ANN strongly suggests that the ML-based algorithm is uniquely adept at modeling the underlying non-linearities present in the Renata Pharmaceutical’s closing price series.

3.2 Simulation Studies for DTMS data

Simulation studies were conducted to evaluate the forecasting performance of different time-series, ML and hybrid models for forecasting damped trend with multiplicative seasonality data in multiple scenarios. Forecasting accuracy was measured by MSE, RMSE, MAE, MAPE, and MASE metrics. To generate the damped trend with multiplicative seasonality time-series data we considered an exponential smoothing state-space model having the structure equation (11).

$$\begin{aligned} y_t &= \left(l_{t-1} + \phi b_{t-1}\right) s_{t-m}\left(1+\varepsilon_t\right) \\ l_t &= \left(l_{t-1} + \phi b_{t-1}\right)\left(1+\alpha\varepsilon_t\right) \\ b_t &= \phi b_{t-1} + \beta\left(l_{t-1} + \phi b_{t-1}\right)\varepsilon_t \\ s_t &= s_{t-m}\left(1+\gamma\varepsilon_t\right), \end{aligned} \tag{11}$$

where l_t, b_t and s_t are level, trend and seasonality equations, respectively, and α, β, γ and ϕ for level, trend, seasonality and damped trend parameters, respectively. Moreover, we assume multiplicative errors with $\varepsilon_t \sim \text{NID}\left(0, \sigma^2\right)$. To generate the required time-series data, we generated normally and independently distributed with mean zero and unit variance, i.e., $\varepsilon_t \sim \text{NID}\left(0,1\right)$ and $\hat{\alpha} = 0.35, \hat{\beta} = 0.001, \hat{\gamma} = 0.005$ and $\hat{\phi} = 0.03$ for sample size $n = 100$, 500, and simulation size $m = 100$. Figures 7 and 8 display time plots of ten example realizations of the simulated DTMS series for sample sizes 100 and 500, respectively. These plots illustrate the characteristic features of the data, including the damped trend and multiplicative seasonal patterns, highlighting both the variability and complexity of the generated series across different sample sizes. The longer series in Figure 8 (sample size 500) show smoother seasonal cycles and trend behavior, as expected due to the larger data length, compared to the shorter series in Figure 7.

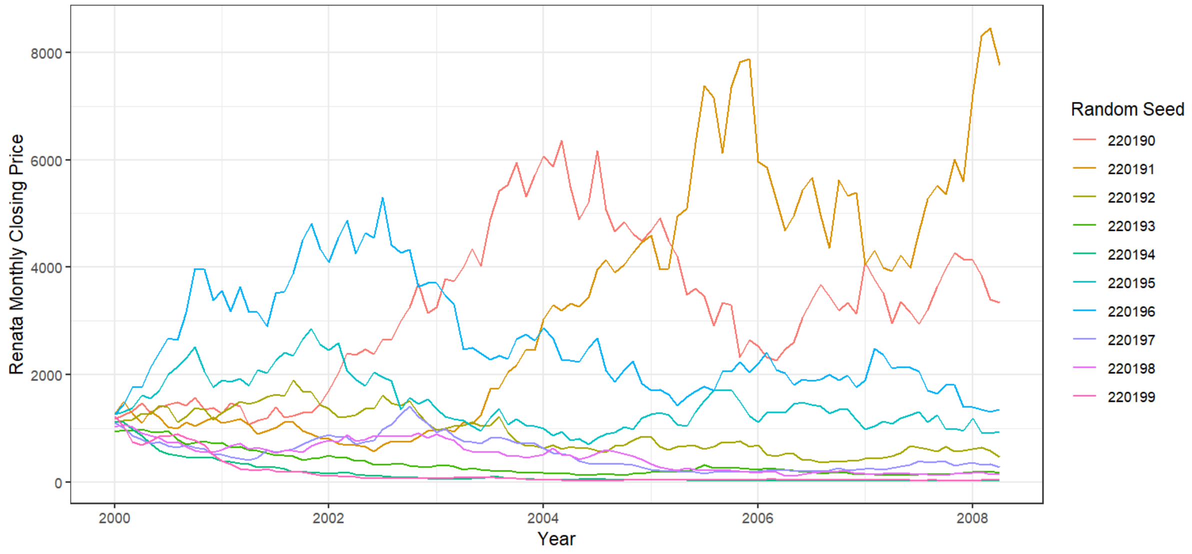

Figure 7: Time plots of ten realizations of the simulated multiplicative damped trend with multiplicative seasonality time-series data starting at random seed 220190 for sample size 100.

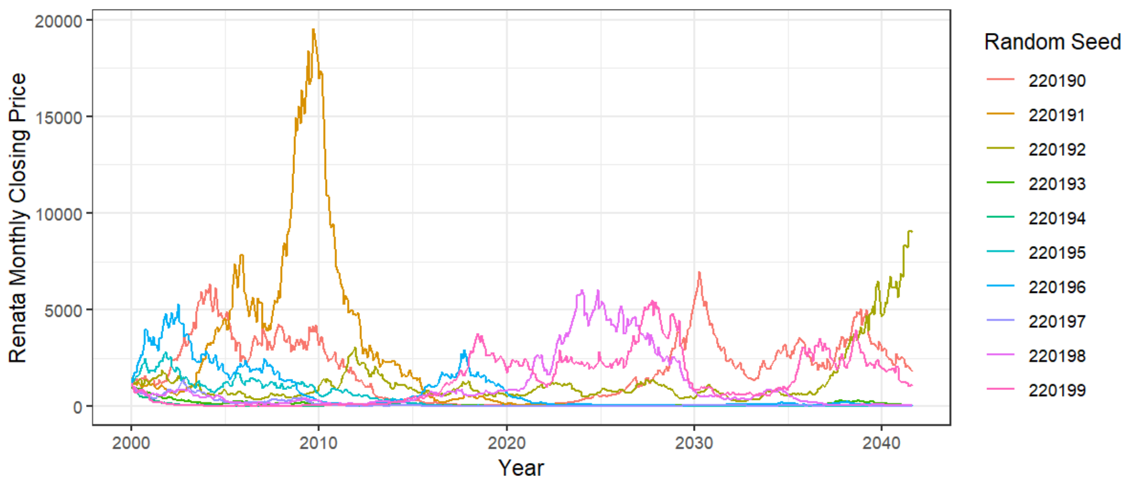

Figure 8: Time plots of ten realizations of the simulated multiplicative damped trend with multiplicative seasonality time-series data starting at random seed 220190 for sample size 500.

Table 2 presents the estimated forecasting accuracy metrics of various classical time-series, ML, seasonal-adjusted, and ML-based hybrid models applied to the simulated DTMS time series data. The results summarize the mean values and their corresponding standard errors (SE) across 100 simulation runs with a sample size of 100. Among the considered models, the hybrid models combining SVR with ETS or ANN consistently delivered superior forecasting performance. Notably, the SVR-ETS and SVR-ANN hybrids achieved the lowest MSE (127,850.45), RMSE (217.86), MAE (158.64), MAPE (18.52%), and MASE (0.701), with relatively low standard errors, indicating both accuracy and stability in predictions. In contrast, traditional models such as ETS, SARIMA, and TBATS showed higher error metrics, reflecting comparatively poorer performance on the simulated DTMS data.

Table 2: Estimated forecasting accuracy measures of different classical time-series, ML, seasonal-adjusted, and ML-based hybrid models for DTMS time series based on simulations $m = 100$, and data sizes $n = 100$.

Model	MSE Mean (SE)	RMSE Mean (SE)	MAE Mean (SE)	MAPE Mean (SE)	MASE Mean (SE)
ETS	402873.85 (1170435.10)	399.57 (495.46)	345.04 (435.00)	46.623% (65.884%)	1.524 (0.883)
SARIMA	495665.26 (1519251.33)	442.16 (550.33)	382.83 (487.00)	61.222% (91.979%)	1.691 (1.012)
TBATS	510751.60 (1621280.24)	427.18 (575.63)	367.21 (498.46)	47.9896% (67.843%)	1.623 (0.945)
ANN	563392.93 (1530163.17)	487.31 (573.76)	424.06 (502.90)	59.800% (76.277%)	1.874 (1.123)
SVR	322590.12 (1480349.78)	293.40 (488.66)	234.37 (410.52)	31.670% (45.411%)	1.036 (0.712)
STL-ARIMA	519276.50 (1420597.92)	467.13 (551.17)	431.12 (657.27)	67.171% (90.472%)	1.905 (1.254)
STL-ETS	455338.66 (1200259.93)	429.31 (523.03)	330.30 (430.09)	50.609% (66.633&)	1.460 (0.821)
STL-SVR	295344.81 (1178503.68)	288.82 (462.58)	227.31 (373.93)	30.800% (45.265%)	1.005 (0.698)
STL-TBATS	705996.72 (4800684.53)	403.63 (740.46)	346.04 (666.50)	41.451% (47.012%)	1.529 (1.102)
STL-ANN	699080.94 (2233128.21)	483.35 (685.40)	421.55 (612.34)	53.961% (64.778%)	1.863 (1.145)
ARIMA-SVR	379830.34 (1056476.15)	387.96 (481.09)	315.42 (425.88)	33.931% (26.141%)	1.394 (0.802)
ETS-SVR	709707.02 (2288296.12)	483.52 (692.93)	433.49 (667.11)	54.547% (67.010%)	1.915 (1.311)
TBATS-SVR	377990.27 (1053685.63)	387.34 (479.66)	330.35 (429.98)	34.044% (26.241%)	1.460 (0.815)
ANN-SVR	455338.66 (1200259.94)	429.32 (523.04)	330.30 (430.10)	50.610$ (66.632%)	1.460 (0.821)
ETS-ARIMA	415642.89 (1254874.55)	408.55 (512.45)	352.11 (445.22)	48.771% (68.116%)	1.556 (0.895)
TBATS-ARIMA	901131.58 (4610423.26)	532.12 (786.15)	465.56 (685.58)	51.368% (35.843%)	2.057 (1.445)
ANN-ARIMA	625487.44 (1781245.87)	518.77 (625.44)	458.33 (565.33)	62.881% (79.452%)	2.025 (1.245)
SVR-ARIMA	335487.55 (1102457.88)	365.45 (475.11)	295.45 (405.45)	32.556% (28.455%)	1.306 (0.754)

Model	MSE Mean (SE)	RMSE Mean (SE)	MAE Mean (SE)	MAPE Mean (SE)	MASE Mean (SE)
ARIMA-ETS	425187.44 (1287541.87)	415.88 (525.11)	360.45 (452.33)	49.129% (69.452%)	1.593 (0.912)
SVR-ETS	**127850.45 (358745.63)**	**217.86 (285.12)**	**158.64 (212.55)**	**18.524% (22.887%)**	**0.701 (0.452)**
TBATS-ETS	377990.28 (1053685.63)	387.35 (479.67)	330.35 (429.99)	34.041% (26.240%)	1.460 (0.815)
ANN-ETS	585214.77 (1687542.33)	495.12 (595.45)	432.15 (535.12)	58.451% (72.330%)	1.910 (1.155)
ARIMA-ANN	598774.55 (1721456.88)	502.45 (605.18)	441.87 (545.87)	60.123% (75.142$)	1.953 (1.187)
ETS-ANN	585214.77 (1687542.33)	495.12 (595.45)	432.15 (535.12)	58.453% (72.335%)	1.910 (1.155)
TBATS-ANN	612335.12 (1754871.45)	510.88 (615.24)	450.24 (555.24)	61.452% (77.886%)	1.990 (1.214)
SVR-ANN	**127850.45 (358745.63)**	**217.86 (285.12)**	**158.64 (212.55)**	**18.524% (22.887%)**	**0.701 (0.452)**
ARIMA-TBATS	875124.55 (2954871.88)	572.45 (845.12)	495.45 745.12)	67.450% (88.122%)	2.189 (1.587)
ETS-TBATS	825487.44 (2854874.12)	555.12 (825.44)	485.12 (725.44)	65.129% (85.440%)	2.144 (1.512)
SVR-TBATS	395124.88 (1154874.21)	398.77 (501.22)	340.18 (445.18)	35.120% (30.111%)	1.503 (0.865)
ANN-TBATS	945787.11 (3254877.45)	595.45 (885.44)	525.45 (785.44)	72.154% (92.452%)	2.322 (1.654)

SE is the standard error of the forecasting accuracy measure across 100 simulations.

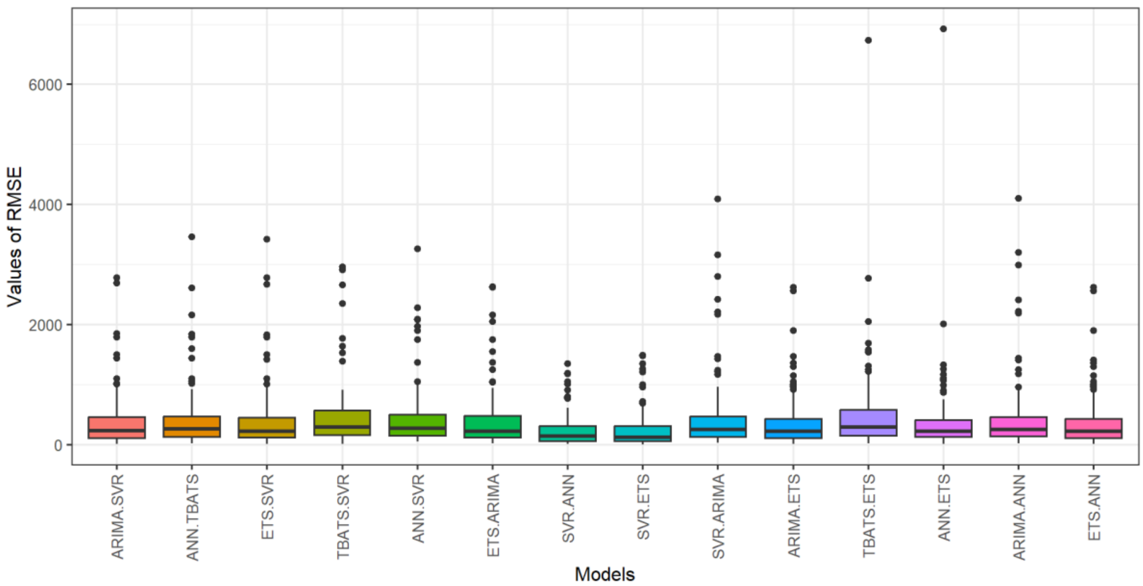

Figure 9: Boxplot of evaluation metric RMSE for forecasting DTMS simulated time series data based on 100 simulation run for sample size 100.

Figure 9 illustrates a boxplot of the RMSE distribution for the forecasting results across the 100 simulation runs. The boxplot visually confirms the superior and consistent performance of the SVR-based hybrid models, which exhibit notably lower RMSE values and tighter interquartile ranges compared to other models. This dispersion pattern emphasizes the reliability of the SVR-ETS and SVR-ANN hybrids in capturing the complex DTMS structure, outperforming both standalone classical and ML models. Hence, these results demonstrate that hybridizing classical exponential smoothing techniques with modern machine learning approaches, particularly SVR, significantly enhances forecasting accuracy for time series exhibiting DTMS.

Discussion

Addressing the significant forecasting challenge presented by time series exhibiting a DTMS was the goal of this study. Through a comprehensive empirical analysis of a real-world financial dataset and an extensive simulation study, we evaluated a wide array of models, from classical statistical methods and ML algorithms to their novel hybrid combinations. The results consistently and unequivocally demonstrate the superior performance of hybrid models that integrate SVR with other techniques, specifically SVR-ANN and SVR-ETS. On the real-world dataset of Renata Pharmaceuticals PLC's stock price, the SVR-ANN and SVR-ETS hybrids achieved the lowest error metrics across all measures (RMSE, MAE, MAPE, MASE). Most notably, their MASE values of approximately 1.07 and 1.08 indicate that they forecast more accurately than a naive seasonal benchmark, a gold standard in forecasting evaluation. Similar findings also reported in [24]. This superiority was not an isolated phenomenon for this study. The simulation study, designed to systematically assess performance under controlled DTMS conditions, robustly confirmed this result. The SVR-ANN and SVR-ETS hybrids again delivered the best performance, achieving a mean MASE of 0.701, which signifies a nearly 30% improvement over the benchmark.

The dominance of these particular hybrids can be attributed to an effective synergy between model components. We hypothesize that the first-stage model (whether ETS or ANN) effectively captures the broad linear and seasonal patterns inherent in the DTMS structure [5,10]. However, these models leave behind complex, non-linear residuals that they are ill-equipped to handle. This is where SVR excels. Its strength in mapping non-linear relationships via the kernel trick allows it to model these residual patterns with high precision [25,26]. The subsequent aggregation of the first-stage forecast and the predicted residuals yields a final forecast that comprehensively captures both the linear and non-linear dynamics of the series. Conversely, the generally poor performance of standalone classical models (ETS, SARIMA, TBATS) and seasonally-adjusted models (STL-ARIMA, STL-ETS) underscores their limitation in handling the non-linear intricacies of DTMS data [20]. While they can capture the trend and seasonality, they fail to model the complex interactions between these components. Similarly, standalone ML models like ANN performed poorly, likely due to their tendency to overfit or their difficulty in learning clear temporal structures without guidance [27]. The dramatic failure of certain hybrids (e.g., STL-ANN, STL-SVR) further highlights that an incorrect model sequence or the loss of information through preprocessing can be detrimental, reinforcing that not all hybridizations are beneficial.

This study advocates for a paradigm shift in forecasting complex time series, integrating classical and machine learning algorithms. The SVR-ETS or SVR-ANN hybrid model, which combines ANN's or ETS's interpretability and SVR's predictive power, is particularly insightful [7]. From a practical standpoint, this research provides practitioners with clear, evidence-based guidance. For forecasting tasks involving DTMS patterns common in finance, economics, and supply chain management the recommended approach is to employ a hybrid model where ETS or ANN are used in the second stage to model the residuals of a primary model. Our results suggest that SVR is an excellent choice for the first stage [25,26]. This strategy offers a robust and powerful alternative to struggling with the limitations of standalone models.

Despite the robust findings, this study has certain limitations that pave the way for future research. First, the real-data analysis was conducted on a single financial time series. While the simulation study adds generalizability, applying this framework to DTMS series from other domains (e.g., energy demand, retail sales) would strengthen the external validity of our conclusions [26]. Second, the hybrid framework presents a two-model combination, requiring further exploration of complex ensembles and stacking models, as well as the use of powerful ML algorithms like gradient boosting machines (e.g., XGBoost, LightGBM) or recurrent neural networks (e.g., LSTM) [28,29]. Future research could focus on developing more efficient automated model selection algorithms to identify the optimal hybrid pairing for a given series without the need for an exhaustive search [30].

Conclusion

The study developed a forecasting framework for time series with damped trend and multiplicative seasonality. It found that hybrid models, SVR-ANN and SVR-ETS, outperformed classical and ML models. This is due to their ability to decompose time series components and address the DTMS scenario, resulting in more accurate and robust forecasts. The research advocates for a hybridized paradigm that uses statistical time series techniques and ML algorithms, which is crucial for accurately predicting complex temporal dynamics. Future research could integrate additional ML models and explore adaptive mechanisms to enhance forecast precision and operational applicability.

Declarations

Availability of data and materials: The datasets that support the findings of this study are available on request. All correspondence and requests for materials should be sent to MKH.

Funding Information: There is no funding for this work.

Authors' Contributions: MKH contributed in processing, and analyzing data, and interpreting the results, editing, and drafting the manuscript. ABA was responsible for data visualization, simulation analysis, and result interpretation. KMZI and RR jointly conceptualized the research idea, design, and supervised the study. Every author has given their approval to the final version.

Conflict of Interest

The authors declare that they have no competing interests.

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